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Variation Theory Based GeoGebra Assisted Problem-solving Approaches and Students' Conceptual Understanding in Learning of Geometry

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Abstract

The study aimed to examine students' problem-solving skills through mixed instruction (variation theory based GeoGebra assisted problem-solving instructional approaches). A total of 125 students divided into four intact groups participated in the study. The study employed a quasi-experimental research design. Three intact groups were randomly assigned as a treatment group, while one group was taken as a comparison group. Each of the groups took a specific instructional approach, while the comparison group proceeded as usual without any changes to the instructional process for all sessions. Both pre and post problem-solving tests were administered to all groups. To analyze the data and examine the differences (if any) in each group, ANCOVA and Paired samples t-tests were employed. There was a significant mean difference between student's pre-test and post-test in their conceptual understanding of each treatment group. Furthermore, the mixed treatment had a large mean difference. Teachers are recommended to give attention in using mixed teaching approaches of variation theory with geogebra assisted problem solving for students' better understanding. Administrators should emphasize launching GeoGebra software through IT labs in schools, and government officials should appreciate the implementation of technology in schools.

Keywords GeoGebra, learning geometry, problem-solving approaches, variation theory Conceptual understanding.

1. INTRODUCTION

1.1 Background of the Study

21st century skills are needed to support us to live prosperously and to achieve better education. The International Society for Technology in Education (ISTE) has recognized problem-solving as a critical educational technology standard essential for success in the 21st century [1]. Problem-solving skill is the capacity to get its issues and steps that will influence students mathematics learning comes about. In education, developing mathematical problem-solving skills serves as a tool not only to help students develop their thinking ability but also to help them build basic skills for solving problems, especially those encountered in daily life. The goal of teaching mathematics effectively is to enable students to solve mathematical problems. In fact, the experience in solving problems in the subject is crucial for developing students' thinking skills and equipping them with essential problem-solving abilities applicable in daily life [2].

As technology continues to influence education, 21st-century learners are increasingly accustomed to a pictorial culture of learning mathematics and other subjects through extensive access to internet resources and various educational software's related to mathematics were available. While this technological influence cannot be avoided, it highlights the shortcomings of traditional lecture-based method of teaching mathematics, which risk becoming outdated. A longstanding problem in mathematics instruction is the lack of adequate visual and illustrative materials to support learning [3].

According to Moges [4], the lecture-based method of instruction remains the dominant practice in Ethiopia's education system. The findings reveal that the integration of educational technology in Ethiopian schools is still at an early stage, mainly due to limited information and communication technology (ICT) infrastructure and a lack of trained teachers. In the United Nations E-Government Development Index (2018) Ethiopia ranked 151st out of 193 countries, underscoring [5] its minimal progress in technological advancement.

Consequently, the Ministry of Education [6], in its Education Sector Development Program VI (ESDP VI) recommends establishing robust system for education data management, assessments, examination procedures, school inspection procedures, and teaching and learning methodologies to ensure effective lesson delivery and an efficient and effective educational system through the use of digital technology. While the implementation of ICT in classrooms aims to improve student learning, numerous administrative and technological challenges hinder Ethiopia's initiative to improve learning through technology [7].

Using variation theory in mathematics instruction can significantly aid in developing students' conceptual understanding. This approach emphasizes the relationship between the object of learning and the students' experiences, empowering them to create a meaningful framework that renders learning significant and applicable. According to [8], students' comprehension improves when they can connect the learning question to their daily lives. This connection fosters a deeper understanding, enabling them to respond effectively to learning challenges. A pivotal aspect of variation theory is the pertinence structure, which bridges students' experiences with the learning question. It clarifies how the learning process shapes students' perspectives, thoughts, and conceptual understanding of topics such as geometry. To ensure effective learning, students' attention must remain on the critical elements of the learning problem being addressed [8]. Integrating technology into teaching and learning further enhances this process. It offers numerous advantages, such as providing broader learning opportunities for students [9] and increasing student engagement [10].

Students' ability to visualize, build, and comprehend shapes plays a vital role in teaching and learning mathematics, particularly geometry, as it enables them to make meaningful connections between shapes and their associated properties. A previous study by [11] highlights the advantages of computer-based tools in fostering creativity and enhancing observational skills among students. Similarly, [12] reported that students in an experimental class using a GeoGebra-assisted problem-based learning approach demonstrated a positive attitude toward learning mathematics, as revealed by the results of an attitude scale.

Similarly, a significant gap exists in linking the effect of GeoGebra software with established learning theories. Given its significance in influencing the learning process, this study adopts an integrated approach that combines GeoGebra with variation theory. This approach aims to address instructional gaps, thereby enhancing the effectiveness of mathematics learning. Accordingly, the purpose of this study is to investigate the impact of variation theory-based GeoGerba-assisted problem-solving approaches on grade 9 students’ learning of geometry.

1.2 Statement of the Problem

Currently, students’ achievement in mathematics in Ethiopia has been decreasing over time, posing a critical challenge to the country’s educational system [12], [13]. Several factors contribute to this decline, including the lack of interrelated content, reliance on conventional (teacher-cantered) methods, limited conceptual understanding, and deficiencies in problem-solving skills, motivation, and attitude [14], [15]. This trend is particularly concerning given that the 1994 Education and Training Policy advocates for a learner-centered approach, which is further emphasized in various policy documents and guidelines.

The Ethiopian National Learning Assessment highlights the issues, with the majority of students unable to score 50% or above, even after completing grade cycles as projected by the Ministry of Education. For instance, the fifth National Learning Assessment reported a score of 35.2% in mathematics for grade 8 students. This score not only falls below the 50% achievement threshold set by the Education and Training Policy of Ethiopia [16] but also shows a consistent decline in performance over the years. Furthermore, it is the lowest compared to other subjects, underscoring students’ inability to attain essential minimum competencies [17]. Similarly, results from the National Learning Assessment, conducted in April 2009, revealed that students fail to meet the required competencies. Different findings from Ethiopia’s education roadmap further emphasize the need to prioritize technology-enhanced learning at all educational levels. The integration of contemporary technological innovations has the potential to address certain challenges in education by fostering the development of conceptual understanding, enhancing problem-solving abilities, and increasing student motivation and engagement [18]. To tackle these pressing issues and align with contemporary educational advancements, this study proposes the use of instruction based on variation theory, supported by GeoGebra software.

1.3 Research Questions

The main objective of this study is to explore the effect of variation theory-based GeoGerba-assisted instructional approaches on 9th-grade students' conceptual understanding of geometry. Based on the objective, the study seeks to address the following research questions:

1. Are there significant differences in the mean gains of the pre-test and post-test of 9th-grade students' conceptual understanding of learning geometry across the three intervention groups and the control group?
2. Are there significant differences in students' conceptual understanding (CU) of learning geometry among the three treatments groups and the comparison group?
3. What challenges are encountered in developing CU when learning geometry through variation theory and computer-assisted problem-solving approaches?

2. REVIEW LITERATURE

The shift from traditional curriculum-based and book-centered learning to more student-driven approaches can be facilitated by incorporating digital technology in the classroom. However, research on the use of digital technology in Ethiopian secondary schools reveals several challenges. Teachers' proficiency in ICT is generally lacking, and there is insufficient training in this area [19]-[21]. Key obstacles include limited ICT infrastructure, inadequate teacher and student ICT skills, insufficient time allotted for in-service and continuing professional development (CPD) courses, limited funding for CPD training, inadequate computer facilities, and poor technical support during teaching. In addition, lack of internet connectivity and backup during power outages in rural areas, and inadequate school policies are some of the major obstacles schools face when it comes to ICT use and integration in education.

The study by [22] recommends the use of mathematical software to encourage inductive reasoning among geometry students in grades 9–12 (SS1-3). Such software can enable students to formulate hypotheses and test their attempts at two-dimensional visualization in an exciting environment created by computer graphics tools, allowing them to create and manipulate shapes.

According to [23], a geometric sketch pad delivers a mathematical laboratory with a flexible structure that facilitates concept exploration and investigation at the representational level, bridging the concrete and the abstract. Analysis of study findings found that when mathematics is taught with the goal of completing the syllabus and getting students ready for exams, many students struggle to understand what their math teachers are teaching.

In addition to using technology, there is advocacy for the implementation of variation theory for better student CU. The study of [24] highlights that procedural variation helps learners acquire knowledge step-by-step, progressively

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develop problem-solving skills, and form well-structured knowledge. Furthermore, students' attention should continually focus on the critical features of the object of learning [8]. Similarly, the study of [25] confirms the effectiveness of variation theory in improving academic achievement.

In 2021, [26] noted that GeoGebra software provides students with a virtual display that simulates a real visual environment, making geometric concepts easy to understand. In addition, compared to learning without geometry, geometry helps students comprehend geometry concepts more fully [27]. This finding was further supported by other research, which revealed that students' mathematical comprehension was significantly enhanced when using GeoGebra software to learn geometry. Based on these results, it can be concluded that using GeoGebra software plays a major and beneficial role in helping students grasp mathematical concepts, particularly in geometry.

With regard to the aforementioned findings, some studies disagree with the researcher's conclusions, while other findings confirm them [28]. The pilot study's analysis also revealed that students' comprehension of the key concepts of the learning object—the relationships between rectangles' area and perimeter—improved significantly when problem-based tasks featuring patterns of variation and invariance were used.

Similarly, [29] contends that using variation theory directs educators toward appropriate pedagogical design that can assist students in identifying the learning objective and produce successful outcomes.

Variation theory is grounded on the concept that individuals become aware of a phenomenon through its variations, either in relation to its environment (external horizon) or in how its internal components vary in relation to one another (internal horizon) [30]. When considering the nature of variations, there are those that, on the surface, seem obvious and those that highlight significant details.

These variations aid in identifying a phenomenon, but there are also differences in how individuals perceive or become aware of it. In phenomenographic studies, these variations are used to develop categories of description. Research has shown that there are a limited number of ways in which people can perceive a phenomenon [30], [31], and these categories are usually arranged hierarchically, with each higher layer incorporating or expanding on the previous one. It is argued that learning occurs when an individual perceives a phenomenon in a new way.

In this study, social constructivism will be applied by encouraging student autonomy and initiative, and fostering discussion even during lectures. The teacher will ask open-ended questions and allow time for students' responses. The students are responsible for creating a collaborative environment focused on problem-solving that encourages them to be active participants in their own learning. Teachers will guide students to develop personal agency and take ownership of their learning. From the social constructivist perspective, it is essential to consider the background and culture of the learner throughout the learning process, as these factors shape the knowledge and understanding the learner creates, discovers, and attains during the learning process [32].

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3. Methods of the Study

3.1 Research Design

Educational research often faces challenges in implementing true experimental designs due to difficulties in fully controlling variables and randomizing participants. As a result, the nonequivalent quasi-experimental design has emerged as a variable alternative. The primary reason for using a nonequivalent quasi-experimental research design with pre-test and post-test is that students in schools are organized into groups at the beginning of each academic year by school administrators, making random assignment into groups, as is required in true experiments, not feasible [33].

According to [34], when full control over the scheduling of experimental stimuli is not possible, such a situation can be regarded as a quasi-experimental design. In the case of nonequivalent groups, participant characteristics may not be equally balanced between the experimental and control groups, which can lead to differences in participants' experiences during the study. For this reason, a nonequivalent control group quasi-experiment design is considered appropriate for this research.

The research design for this study involves one comparison group and three treatment groups, each with pre-test and post-test assessments. The treatment groups will receive different instructional approaches: Treatment Group 1 (TG1) will be taught using Variation Theory-based GeoGebra instructional approaches; Treatment Group 2 (TG2) will follow Variation Theory-oriented instructional approaches; and, Treatment Group 3 (TG3) will employ GeoGebra-assisted instructional approaches. The comparison group will receive traditional instructional methods.

TABLE I. Classification of Groups for the Treatments

Groups	Pre-test	Treatment	Post test
Treatment Group 1 (TG1)	O_1	X_1	O_2
Treatment Group 2 (TG2)	O_1	X_2	O_2
Treatment Group 3 (TG3)	O_1	X_3	O_2
Comparison Group (CG)	O_1		O_2

3.2 Population and Sampling

The study was conducted in Addis Ababa, Ethiopia, with grade 9 students. The choice of grade 9 is significant because secondary education in Ethiopia begins at this grade, which serves as the foundation for subsequent grade levels. Success in higher education depends on the quality of secondary education. This level of education, therefore, needs to equip learners with the necessary skills and specific knowledge that serve as a base for higher education.

Four schools were selected using simple random sampling after selecting the sub city by lottery method for the intervention and comparison group. Although nonequivalent quasi-experimental research design inherently involves differences between groups, according to [35], the researcher should select groups that are as similar as possible to avoid bias. The aim of selecting different schools was to avoid contamination of information during the intervention and data collection phase. After selecting schools, each of the four schools was assigned to either the treatment or comparison group by using a simple random sampling method (lottery).

For each of the four schools, one mathematics teacher and one section of grade nine students participated in the study, with participants selected using the lottery method. The teachers involved in the study had taught grade nine mathematics in the academic year and possessed similar qualifications and experience in teaching grade 9 mathematics.

3.3 Data Collection Instrument

Data were gathered using various data gathering tools, including a CU test, interviews, and classroom observations. The CU test was designed to measure individual knowledge or skills in mathematics. Geometry questions grade 9 were used for the post-test, and geometry question for grade 8 were used for the pre-test. The tests, consisting of 10 questions, were selected from the Ethiopia General Secondary Education Certificate Examinations (EGSECE) and the Addis Ababa Educational Bureau regional exams for grade 10 and grade 8 students before 2011 E.C., covering five consecutive years. Since the EGSECE contains grade 9 questions, these were also relevant. To ensure the accuracy of the questions, four subject teachers from each school involved in the study validated them.

The conceptual understanding test (CUT) questions were administered twice: as a pre-test before the intervention and as a post-test after the intervention. The CUT was a two-tier multiple-choice test. The first tier consisted of multiple-choice content responses, offering four multiple choices, while the second tier provided a set of four to five possible reasoning responses, along with additional blank space to diagnose students' CU.

3.4 Validity and Reliability of the Instrument

In this study, content validity was ensured through panel discussions with mathematics teachers and supervisors. According to [35], establishing a panel of experts to review the instrument during development is an effective method for determining validity. Therefore, the content and face validity of the instrument were checked using the opinions of experts in mathematics, psychology, and curriculum and instruction. The CUT was reviewed by experts, including mathematics teachers, PhD candidates in mathematics education, and secondary school mathematics teachers. They evaluated the test items for consistency with text book objectives and concepts, clarity of the items, and any errors in the key answers. Corrections were made based on the feedback and recommendations obtained from the experts.

To assess the practicability of the instruments, as well as the difficulty and discrimination levels of test items and reliability of the instruments, a pilot test

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was conducted at selected schools in Addis Ababa City Administration. The final version of the data collection instrument was piloted with 40 grade 9 students. Based on the results from the pilot, item analysis was performed for the CUTs, using parameters such as difficulty index and discrimination index, and the items were found to was fit within the required range.

The internal consistency of the instrument was also estimated using Cronbach's coefficient alpha, which was above 0.71. After passing all the validity and reliability checks, and making the necessary revisions, the final version of the test was administered as both a pre-test and post-test to assess student's CU before and after the intervention.

3.5 Methods of Data Analysis

Data collected from different sources were analyzed quantitatively using appropriate statistical analysis method. The analysis was conducted using the Statically Package for Social Science (SPSS) version 20 software. Since the data were normally distributed, parametric statistics were employed, as the data met the assumptions required for parametric tests. Therefore, mean, standard deviation, paired t-test, ANOVA or ANCOVA, were utilized for data analysis. For the qualitative data obtained from the semi-structured interviews, categorization under relevant themes was carried out.

4. RESULTS AND DISCUSSIONS

4.1 Results

Are there significant differences in the mean gains of the pre-test and post-test of 9th grade students' CU of learning geometry within the three intervention and control groups?

In order to answer the question above, we first check whether there are significant differences in the mean of pre-tests of students' problem-solving skills in learning geometry among the three interventions (variation theory-based geometry problem-solving instructional approaches; variation theory-based problem-solving instruction; GeoGebra-assisted instructional approach) and control groups. In order to answer the above question the ANOVA test was employed. To do so, the assumption of an ANOVA was fulfilled. These are: the data are continuous and normally distributed; the data are independent since the intervention was conducted in different schools, where there is no chance for the respondent to communicate with each other; and the dependent variable has the same variance across groups.

TABLE II. THE DESCRIPTIVE STATISTICS OF PRE-TEST OF CU IN LEARNING GEOMETRY SCORES BY GROUPS

Dependent Variable	Groups											
	TG1			TG2			TG3			CG		
	N	M	SD	N	M	SD	N	M	SD	N	M	SD
Pre- CU	34	6.82	4.34	35	5.77	3.47	29	7.04	3.75	27	5.41	2.99
Post- CU	34	14.53	2.9 2	35	8.37	3.32	29	8.89	2.71	27	6.11	3.02

Table II shows the mean Pre-geometry CU scores for intervention group 1, intervention group 2, and intervention group 3 where M = 6.82, M = 5.77, and M = 7.03, respectively, while M =5.41 for the comparison group. The table also indicates that the mean post-geometry CU scores for intervention group 1, intervention group 2, and intervention group 3 were M = 14.53, M = 8.37, and M = 8.89, respectively, while M = 6.11 for the comparison group.

In order to check the difference between the groups and their CU of learning geometry before the intervention the following table illustrates.

TABLE III. ANOVA TEST ON PRE-TEST OF STUDENTS CU IN LEARNING GEOMETRY SCORES BY GROUPS

Variables		Sum of Squares	df	Mean Square	F	p
Pre-geometry CU	Between Groups	56.155	3	18.718	1.542	.207
	Within Groups	1468.597	121	12.137		
	Total	1524.752	124			

As can be seen in Table III, the results of the study show that F (3, 121) = 1.542, p = 0.207 for Pre-geometry CU in learning geometry scores between the groups. This indicates that there was no statistically significant mean difference between the four groups regarding their pre-test CU result since the p value > 0.05.

Now it is time to check whether there are significant differences in the mean gains of the pre-test and post-test of students' problem-solving skills in learning geometry among the three intervention and control groups.

Assumptions on a Paired T-Test

1. The free variable is dichotomous, and its levels (or groups) are matched, or coordinated, in a few way..

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2. The dependent variable is normally distributed, as indicated above.

Therefore, Table IV shows the difference between students CU before and after the intervention.

TABLE IV. PAIRED SAMPLES T-TEST ON MEAN GAINS OF THE PRE-TEST AND POST-TEST OF STUDENTS' CU IN LEARNING GEOMETRY SCORES BY GROUPS

Mean Gain	Group	Paired Differences		t	df	p
		M	SD			
Geometry: (Pre –CU)-(Post-CU)	Treatment group -1	7.706	4.303	10.442	33	.000
	Treatment group -1	2.6	3.136	4.905	34	.000
	Treatment group -1	1.862	3.523	2.847	28	.008
	Comparison Group	.704	2.614	1.399	26	.174

Table IV presents data showing the mean difference in students' CU from pre-test to post-test across different groups. In the group that received a variation theory-based geometry problem-solving instructional approach, the mean difference was 7.706 with a standard deviation of 4.30313, t-value of 10.442, df of 33, and a p-value of .000.

For the group given a variation theory-based problem-solving instruction approach, the mean difference was 2.6, with a standard deviation of 3.13613, a t-value of 4.905, a df of 34, and a p-value of 0.000. Similarly, on student's CU of pre-test to post-test in the group taught using a GeoGebra-assisted instructional approach, the mean difference was 0.704, with a standard deviation of 3.522622.847, t-value of 2.847, df of 28, and a p-value of .008. Finally, in students' CU of pre-test to post-test in the comparison group, was 0.704, with a standard deviation of 2.61379, t-value of 1.399, df of 26, and a p-value of .174.

From the data, the mean difference between students' pre-test to post-test scores on their CU indicates a significance difference in each treatment group, as the p-value < 0.005. However, there is no significance difference in the comparison group's pre-test to post-test results on students' CU, with p = 0.742 (which is >.05).

The following research question tries to address whether a significant difference exists between the groups after the interventions, with the pre-test CU observed as a covariant:

Are there significant differences in students' CU of learning geometry among the three treatment groups and the comparison group?

To answer this question, the ANCOVA test was conducted after verifying the necessary assumptions.

The covariates are measured before the intervention to ensure that the scores were not influenced by the intervention, and they are not significantly correlated with one another, as previously mentioned. Furthermore, the covariates were measured consistently. These findings support some the assumptions of ANCOVA.

Other assumptions, such as the independence of observations, normality, homogeneity of variance, and linearity between the covariates and dependent variables at each level of the independent variables, were also satisfied. Table V represents the group means with or without controlling for the covariate(s) (adjusted and unadjusted means).

TABLE V. ADJUSTED AND UNADJUSTED MEANS AND VARIABILITY ON THE THREE GROUPS FOR CU IN LEARNING GEOMETRY USING PRE-TEST AS A COVARIATE

Groups	N	Unadjusted		Adjusted	
		M	SD	M	SD
Treatment group -1	34	14.529	2.915	14.289	.498
Treatment group -2	35	8.371	3.317	8.590	.490
Treatment group -3	29	8.896	3.74 5	8.564	.540
Comparison Group	27	6.111	3.017	6.488	.561

Table V presents the mean of the post-test scores for student's CU of learning geometry, showing both the unadjusted and adjusted means. The adjusted mean scores for the groups were as follows: 14.529 for the variation theory-based GeoGebra problem-solving instructional approaches group, 8.371 for the variation theory-based problem-solving instruction group, 8.896 for the GeoGebra-assisted instructional approach group, and 6.111 for the comparison group.
After adjusting for covariates, the mean scores were 14.289, 8.590, 8.564, and 6.488 for the variation theory-based GeoGebra problem-solving instructional approaches group, the variation theory-based problem-solving instruction group, the GeoGebra-assisted instructional approach group, and the comparison group, respectively.
The data indicate minimal differences between the adjusted and unadjusted means for students’ CU of geometry. Figure-1 illustrates these differences more clearly.

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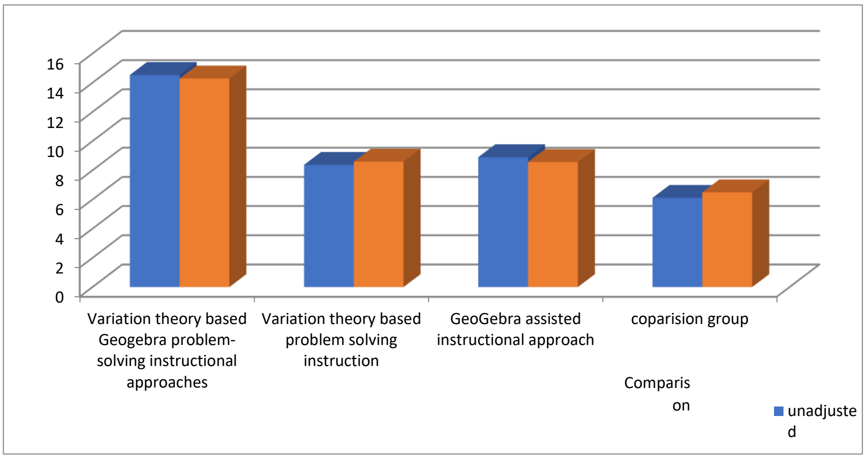

Fig. 1 Unadjusted and adjusted mean for students CU post-test

From Fig. 1, we can observe that the estimated marginal means for all groups were statistically adjusted to enable comparison between the pre-test and posttest scores as well as as among the groups in their post-test. The adjusted post-test mean scores of the students also demonstrate a significant difference in favor of the intervention groups compared to the comparison group, as shown in Table 5 and Fig. 1. This indicates that mean scores improved after applying the variation theory-based geometry problem-solving instructional and the variation theory-based problem-solving instruction approaches. However, this does not confirm pairwise differences or identify the best-performing group.

Given the presence of more than two groups, the Bonferroni test, which is a type of post hoc test, was employed to analyze the adjusted post-test means to determine which group performed better. The Bonferroni test results are reported in Table VII to identify the source of the observed differences.

Table VI provides the results of the analysis of covariance (ANCOVA) for problem-solving skills in learning geometry, using pre-test scores as a covariate.

TABLE VI. TESTS OF BETWEEN-SUBJECTS EFFECTS

Source	df	MS	F	P	Partial eta^2
Pre-CU	1	279.803	33.436	.000	.218
Group	3	353.023	42.186	.000	.513
Error	120	8.368			

As shown in Table VI, when the effect of the students' pre-CU was controlled, a statistically significant difference was observed between the groups' post-CU scores, F (3, 120) = 33.436, p =.000, η^2 = .218. This indicates that the students' post-CU scores were associated with the instructional methods used, even though the pre-test results also significantly influenced the student's post-CU ($p < 0.05$).

In this study, the partial eta-squared value (η^2) for the group is .513, which, according to Cohen's guidelines (1988), represents a medium effect size. This value suggests the instructional approaches accounted for 51.3% of the variability in the dependent variable. This indicates that the teaching approaches thus had a large effect on the groups' post-test mean scores.

The table also highlights that the pre-test results on students' CU in learning geometry had a significant impact, with a partial eta-squared value of 0.218, also indicating a medium effect. It determine which groups showed significant differences, Table VII provides the results of the Bonferroni post hoc comparison analysis.

TABLE VII. GROUP COMPARISON ON STUDENTS POST TEST RESULT

(X) group	**(Y) group**	**Mean Difference (X-Y)**	**P**
TG1	CG	7.800*	.000
	TG2	5.699*	.000
	TG3	5.725*	.000
TG2	CG	2.101*	.032
	TG3	.026	1.000
TG3	CG	2.075	.055

. Dependent Variable: post-conceptual understanding.
* significant difference exist at the .05 level .

The adjusted post-test score results from the Bonferroni test show that the mean difference between TG1 and CG is 7.8, with a p-value 0.000; the mean difference between TG1 and TG2 is 5.699, with a p-value 0.000; and the mean difference between TG1 and TG3 is 5.725, with a p-value 0.000. Similarly, the adjusted post-test score results from the Bonferroni test show that the mean difference

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between TG2 and CG is 2.101, with a p-value 0.032; the mean difference between TG2 and TG3 is 0.026, with a p-value 1.00. On the other hand, the mean difference between TG3 and CG is 2.075, with a p-value 0.055.

The data indicate a statistically significant difference in students' CU between TG1 and the other three groups, as p-value < 0.05. This suggests that TG1 demonstrated better CU while learning geometry compared to the other groups. However, there was no statistically significant difference in students' CU between TG2 andTG3 (p-value > 0.05). Similarly, no statistically significant difference was observed between TG3 and the CG (p-value > 0.05).

Furthermore, the high performance of TG1 is supported by students' ability to reason effectively, as the CUT was two tired, measuring both content knowledge and reasoning. The statistically significant difference in TG1's performance on CU highlights their enhanced reasoning abilities compared to the other groups.

The interaction between the students and the intervention, as well as dynamics between students and teachers during lessons, addressed the following question:

What challenges are there in CU when learning geometry through variation theory and algebra-assisted problem-solving approaches?

During the implementation of variation theory in GeoGebra-assisted problem-solving approaches, teachers encountered several challenges. Based on qualitative analyses of interviews and observation checklists, the study identified the following challenges: overcrowded classrooms, limited access to computer labs (they were occupied by IT classes), students preparedness, difficulty grasping core concepts, teacher skills gaps in implementing the GeoGebra Software, variation theory approaches, and problem-based approaches, as well as curriculum restraints due to the dense content, which covers 7 weeks, often leading teachers to adopt content-heavy, linear teaching methods using school textbooks.

During the observation of sessions, teachers reflected on their experiences, emphasizing that GeoGebra is a valuable tool for supporting mathematics education when used effectively. Similarly, procedural variation was noted to assist students solve problems more efficiently. Some reflections of teachers on the use of GeoGebra and variation theory are presented as follows:

"If I had known GeoGebra prior that would have been great for understanding geometric ideas, concepts are made evident by the software's dynamic nature; now it gives me a good understanding of the idea of similarities and congruence of triangles."

"There is no denying the software's importance; however, the lack of resources and computers in the school presents a challenge. I will go further with the implementation of the software for my other section next."

"When I used variation theory-based GeoGebra-assisted problem-solving approaches, students were interested, but I was worried about whether I covered the lesson on the plane ahead."

Although variation theory, algebra-assisted problem-based learning strategies, and variation theory-based problem-solving approaches support the practical application of mathematical concepts, teachers noted that these approaches are time-consuming, making it difficult to cover the extensive school curriculum. Furthermore, teachers were also challenged by large class sizes in the study area.

These overcrowded classrooms reportedly hindered teachers' ability to organize and manage practical activities effectively, supervise and stimulate small group discussions on practical activities, and summarize discussions efficiently for the entire class when using variation theory, GeoGebra-assisted classroom, and variation theory-assisted problem-solving approaches.

4.2 DISCUSSION

The quantitative and qualitative findings revealed significant differences between the treatment and control groups in the CU of students in learning geometry. Students in the treatment groups demonstrated higher CU in the post-test compared to the control group. These findings indicate that instruction using variation theory-based GeoGebra-assisted instruction, variation theory-assisted instruction, and GeoGebra-assisted instruction can effectively enhance students' CU.

In the case of TG2 (variation theory-assessed instruction), several scholars corroborate its positive impact on students' CU. Repeated visualization of geometric figures helps students develop a deeper CU. During lesson delivery, students expressed amazement at the formation of plane figures. Reference [23] highlights procedural variation facilitates stepwise knowledge acquisition, progressive problem-solving skills, and the development of well-structured knowledge. Furthermore, it emphasizes that students' attention should be directed toward critical features of the object of learning [7]. For students to gain awareness, they must focus on the object against a meaningful background. Similarly, [24] validated the effectiveness of variation theory in achieving better academic outcomes.

In 2021, B. Tamam and D. Dasari highlighted that GeoGebra provides students with a virtual platform simulating a real visual environment, making geometric concepts more comprehensible. They further noted that learning with geometry software, compared to traditional methods, enables students to comprehend geometry concepts more effectively [26]. This finding aligns with other research, which reported a significant positive impact of using geometry software on students' mathematical comprehension [36]. These results collectively conclude that incorporating GeoGebra into mathematics instruction, particularly geometry, significantly improves student's understanding of the material.

While most findings align with the study, some studies offer contrasting views, as noted in [27]. The pilot study also demonstrated that students' comprehension of key concepts—such as the relationship between a rectangle's area and perimeter—improved significantly when problem-based tasks integrating patterns of variation and invariance were employed. This study showed that students' CU improved when lessons utilized variation theory, geometry-assisted problem-solving approaches, or hybrid approaches.

Lessons was combining variation theory with geometry were particularly effective in fostering students' CU. GeoGebra and variation theory, when implemented effectively, significantly enhance students CU. During the initial intervention, teachers faced challenges and confusion; however, with subsequent classes, they adapted well, and students actively participated and were eager to learn.

In line with these findings, [28] contends that variation theory provides educators with a framework for designing effective pedagogical strategies, enabling students to identify learning objectives and achieve successful outcomes.

5. CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

The study aimed to investigate the effect of variation theory-based GeoGerba-assisted instructional approaches on 9th-grade students' CU of geometry. The findings revealed a statistically significant difference between the pre- and post-test mean scores of students' CU within the intervention groups. However, no significant difference was observed in the comparison group. Furthermore, the study found a statistically significant difference in post-test scores among the groups when the effect of pre-conceptual understanding was controlled.

Based on these findings, the following conclusions were drawn:

1. Variation theory instruction had a substantial positive impact on students' CU of learning geometry.
2. GeoGebra contributed significantly to enhancing students' CU.
3. Variation theory-based GeoGebra-assisted instruction proved to be an effective approach for improving students' CU.

5.2 Recommendations

Based on the conclusions and the findings in this study, the following recommendations are proposed:

1. Teachers should adopt variation theory, geometry-based methods, and mixed approaches (e.g., variation theory-based geometry problem-solving approaches) to enhance students' understanding of geometry.
2. School administrations should prioritize the incorporation of GeoGebra software into IT labs and provide resources to support its integration onto the curriculum.
3. Government officials should support and encourage the implementation of technology in schools to align with modern curricula and improve educational outcomes.

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